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A Satellite Navigation Signal Scheme Using Zadoff-Chu Sequence for Reducing the Signal Acquisition Space

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ABSTRACT

A signal system for improving the code acquisition complexity of Global Navigation Satellite System (GNSS) receiver is proposed and the receiving correlator scheme is presented accordingly. The proposed signal system is a hierarchical code type with a duplexing configuration which consists of the Zadoff-Chu (ZC) code having a good auto-correlation characteristic and the Pseudo Random Noise (PRN) code for distinguishing satellites. The receiving correlator has the scheme that consists of the primary correlator for the ZC code and the secondary correlator which uses the PRN code for the primary correlation results. The simulation results of code acquisition using the receiving correlator of the proposed signal system show that the proposed signal scheme improves the complexity of GNSS receiver and has the code acquisition performance comparable to the existing GNSS signal system using Coarse/Acquisition (C/A) code.

Keywords: Zadoff-Chu sequence, GNSS, acquisition search space

1. INTRODUCTION

For calculating the position of receiver using a Global Navigation Satellite System (GNSS) signal, the distance from satellite to receiver and the coordinate of satellite are necessary, and the positioning in 3-dimensional space can be accomplished when there are at least 4 visible satellites. In order to reduce the error of distance measurement from satellite to receiver, the temporal resolution, which is the basis of distance measurement, needs to be increased, and the signal reaching the ground has weaker intensity than thermal noise. In this regard, every GNSS signal system employs the spread spectrum modulation to achieve the increase of temporal resolution for distance measurement and the processing gain.

On the other hand, to distinguish various satellites, the spread spectrum modulation is performed so that the spread spectrum signal generated at each satellite has different sign. When this code for each satellite is received in a mixed form, the code that minimizes the cross-correlation value between the signals is used, and the Coarse/Acquisition (C/A) code of Global Positioning System (GPS) is a representative code for this purpose. Also, for the additional reduction of cross-correlation value, the code period is generally maximized within the limit that minimizes the effect of Doppler shift due to the movements of satellite and receiver. The Doppler shift due to the relative movements of satellite and receiver which is observed at the receiver could be a major factor for degrading the correlation performance of receiving correlator. Therefore, regarding the Doppler shift which is dispersed over a range of more than several kHz, a typical GNSS receiver uses the method that obtains the correlation value by changing the frequency of correlator.

Based on the above discussion, Fig. 1 shows the search range of receiving correlator for the satellite signal acquisition of typical GNSS receiver. The frequency search interval by the Doppler shift is generally 500Hz, and the number of satellites that a GPS has at present is about 30.

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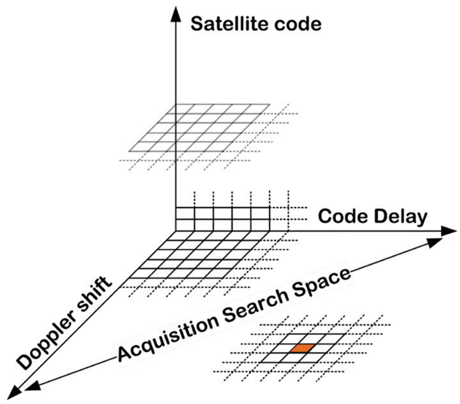

Fig. 1. Acquisition space of 3-dimension.

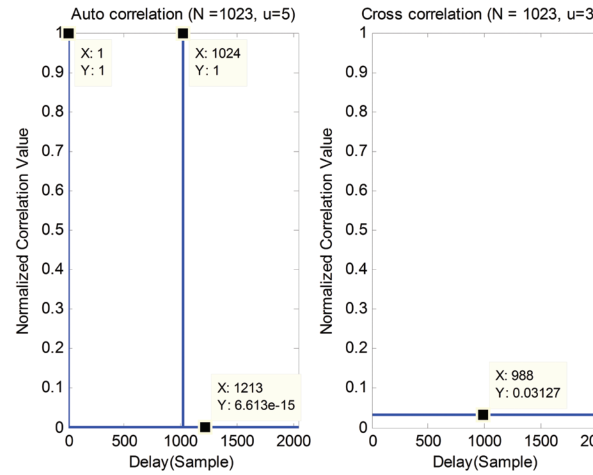

Fig. 2. Property of Zadoff-Chu sequence.

For the acquisition range from the code delay axis, the GPS calculates the correlation values of 1023 chips for each cell. However, for the Gailieo from Europe, the calculation for the signal acquisition increases because the correlation values of 10230 chips need to be considered. As the signal acquisition cell formed by the values assigned to each axis increases, the complexity also sharply increases. The increase in the density of hardware technology is trying to cope with the complexity increase of code acquisition correlator, but this could lead to the power and heat problems of portable device. Thus, it is necessary to conduct studies on complexity reduction.

The purpose of this study was to find a method for reducing the code acquisition complexity of GNSS receiver by improving the signal scheme. In the signal scheme of the existing GNSS, one of the causes for the increase of correlator complexity is that an independent receiving correlator needs to be operated for each satellite. If every satellite generates the spread spectrum signal as the same code but the satellites could still be distinguished, the signal processing procedure for satellite search would be innovatively simplified.

To design such signal system, the Zadoff-Chu (ZC) code having a good auto-correlation characteristic was selected, and this code was assigned to the spread spectrum modulation for every satellite. The ZC code is widely used as the preamble for synchronization for the 4th Generation (4G) systems such as Long Term Evolution (LTE) and Ultra Wide Band (UMB) (Beyme & Leung 2009). In designing the system, the satellites are distinguished by making the GNSS signal to be a hierarchical code type via adding different Pseudo Random Noise (PRN) code to the basic period ZC code for each satellite.

In Chapter 2, a new GNSS signal system which uses the ZC code and PRN code is introduced, and in Chapter 3, the scheme and operation of the correlator for code acquisition which can be applied to the new signal system are described. In Chapter 4, the correlation characteristic of proposed signal system is presented through the simulation, and in Chapter 5, the complexity effect and the conclusion are included.

2. DESIGN OF GNSS SCHEME USING ZC

2.1 Zadoff-Chu sequence

The ZC code which has the root of u and the length of prime number length N_{zc} is defined as Eq. (1).

$$x_u[n] = \begin{cases} \exp\left\{\frac{-j\pi un(n+1)}{N_{zc}}\right\} & (u: \text{odd}) \\ \exp\left\{\frac{-j\pi un^2}{N_{zc}}\right\} & (u: \text{even}) \end{cases}, \quad 0 \le n < N_{ZC} \tag{1}$$

The ZC code has a feature that it does not have a correlation value which excludes the case when the synchronization is matched. An independent code generation is possible depending on the selection of the root u of code which is relatively prime to the code length N_{zc}. Fig. 2 shows the correlation characteristic of ZC code which has the N_{zc} of 1023 that is identical to the C/A code length of GPS. The correlation value is found to be close to 0 for the remaining code delay except for the case that it matches exactly.

2.2 Modulation scheme using Zadoff-Chu sequence

The signal scheme of GNSS should be able to distinguish every satellite and transmit the navigation information that each satellite sends simultaneously. For reducing the complexity, every satellite needs to have the same code,

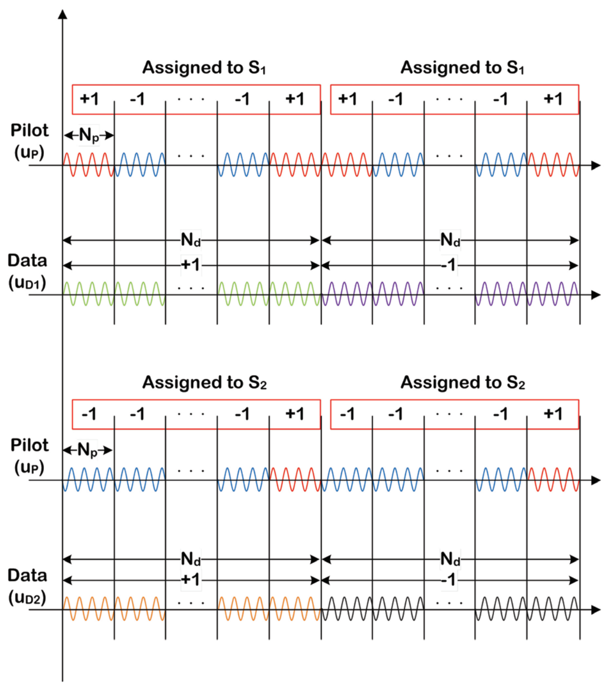

Fig. 3. Satellite signal scheme.

and the code having a good auto-correlation characteristic is more advantageous than the code having a good cross-correlation characteristic. Therefore, using the Zadoff-Chu code having a good auto-correlation characteristic, the signal scheme is proposed as shown in Fig. 3 which is a combined type of the pilot signal for distinguishing satellites and the data signal.

Each satellite repeatedly sends the same ZC pilot code, and the satellites are distinguished by hierarchically assigning the PRN code for every ZC pilot code period. When this scheme is applied, the PRN code for distinguishing satellites could be obtained from the correlation value of pilot signal. This requires the calculation for comparing the PRN codes, but as they are relatively short length codes, the calculation will be insignificant when compared to the existing calculation.

For the data signal which carries the navigation information, in the pilot signal scheme, the navigation information is assigned to the ZC code which has a length that is the multiple of the PRN code length, and is transmitted simultaneously with the pilot signal. Using formulas, this can be defined as Eqs. (2) and (3).

$$Z_p[n] = \begin{cases} \exp\left\{\frac{-j\pi u_p n(n+1)}{N_p}\right\} & (u: odd) \\ \exp\left\{\frac{-j\pi u_p n^2}{N_p}\right\} & (u: even) \end{cases}, \quad 0 \le n < N_p \qquad (2)$$

$$Z_{d,k}[n] = \begin{cases} \exp\left\{\frac{-j\pi u_{d,k} n(n+1)}{N_d}\right\} & (u: odd) \\ \exp\left\{\frac{-j\pi u_{d,k} n^2}{N_d}\right\} & (u: even) \end{cases}, \quad 0 \le n < N_d \qquad (3)$$

As shown in Fig. 3, the length of ZC code consists of the pilot and the data. The data length is equal to the length which added the code length to the pilot length. Then, the N_d, which determines the data length, naturally becomes the multiple of N_p, and if the two lengths are designed to be relatively prime to each other, the polyphase code condition could be satisfied (Chu 1972).

$$S_k^p[n] = \sum_{m=0}^{N_c-1} Z_p[n - mN_p] \cdot (-1)^{C_k(m)+1} \qquad (4)$$

$$S_k^p[n] = S_k^p[n + N_d] \qquad (5)$$

In Eq. (4), for the k-th satellite, the identification number is assigned to the PRN code C_k, and as shown in Eq. (5), this is put in the pilot signal and periodically generated for every data length N_d.

$$S_k^d[n] = \sum_{l=0}^{\infty} Z_{d,k}[n - lN_d] \cdot (-1)^{D_k(l)+1} \qquad (6)$$

$$S_k[n] = S_k^p[n] + S_k^d[n] \qquad (7)$$

The signal sequence D_k, that the k-th satellite sends, is also expressed as Eq. (6) depending on the sign of data, and the transmitting signal of k-th satellite which combines these two signals can be expressed as Eq. (7). The pilot and data signals generated in this way are transmitted in a mixed form, and as the code correlation between the signals is insignificant, it will be possible to distinguish the signals.

3. CORRELATION SCHEME FOR ACQUISITION

3.1 Receive signal model

As for the GPS, for the acquisition of signal, the correlation value is obtained by generating the reference signal that corresponds to each cell shown in Fig. 1 to despread the signal which has been spreaded for about 1ms in the correlator. In the actual receiver environment, the signal arrives while experiencing different delay and attenuation from the satellite that is in operation at each orbit. An ideal receiver without the Doppler effect is

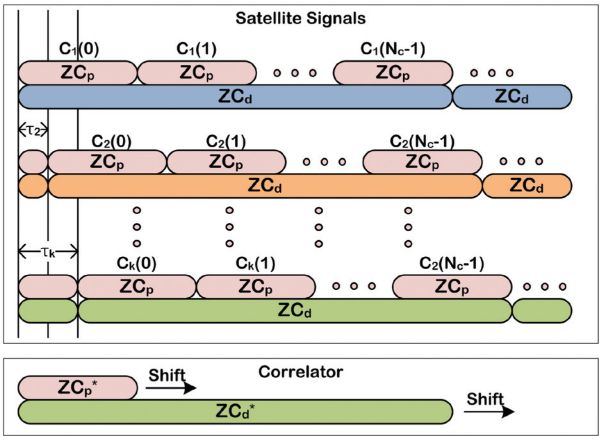

Fig. 4. Despreading for proposed satellite signals.

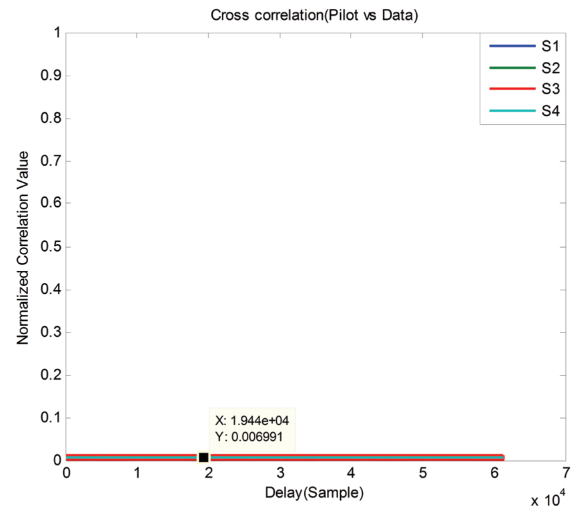

Fig. 5. Interference between data and pilot.

assumed. Based on this, the receiving signal can be modeled as Eq. (8). y[n] is the sum of signals where the k-th satellite has the delay of τ_k and the attenuation index of g_k, among the N_s visible satellites.

$$y[n]=\sum_{k=1}^{N_s} g_k S_k[n-\tau_k] \tag{8}$$

3.2 Correlation model

For the previously modeled satellite signal, to distinguish satellites from the received signal depending on each code delay, the correlation value which uses the pilot signal can be modeled as Eq. (9).

$$R_p[q]=\sum_{n=0}^{N_p\text{-}1}\sum_{k=1}^{N_s} g_k\{S_k^p[n-\tau_k]+S_k^d[n-\tau_k]\}\cdot Z_p^*[n-q] \tag{9}$$

If the correlation value with the data is relatively prime to the pilot signal, the cross-correlation characteristic of certain numerical value will be observed as shown in Fig. 2. The diagram for the receiving signal and despreading procedure depending on time delay is shown in Fig. 4.

$$\begin{aligned} R_p[q] &= \sum_{n=0}^{N_p\text{-}1}\sum_{k=1}^{N_s} g_k S_k^p[n-\tau_k]\, Z_p^*[n-q] + \text{Interference} \\ &= \sum_{n=0}^{N_p\text{-}1}\sum_{k=1}^{N_s} g_k \exp\left\{\frac{j\pi u_p}{N_p}\left(2(q-\tau_k)n+\tau_k{}^2-q^2 +q-\tau_k\right)\right\} + \text{Interference} \end{aligned} \tag{10}$$

In Eq. (10), the interference component is the interference due to the cross-correlation between the data ZC code and pilot ZC code, and is about 38 dB which is insignificant compared to the auto-correlation peak value as shown in Fig. 5. When the pilot channel code delay q and receiving delay τ_k match exactly, every exponential term becomes 0 and the correlation result is expected to have the peak value. When they do not match exactly, it has a linear phase value with respect to n, and the correlation result is expected to be close to 0 depending on the ZC code characteristic.

Therefore, in the receiver, the satellite code could be obtained from the sign of the correlation value of pilot signal. For the data signal, the extraction of data sign would also be possible by obtaining the correlation value, similar to the pilot signal.

4. SIMULATION AND RESULT

For testing the signal scheme proposed in Chapters 2 and 3, by applying the parameters of the environment without noise and Doppler shift assumed in Section 4.1, it is shown, in Sections 4.2 and 4.3, that the satellite could be distinguished and the navigation information could be obtained through the pilot signal and data correlation value. In Section 4.4, the signal power spectrum is analyzed, and in Section 4.5, the applicability of satellite navigation signal scheme is examined by comparing with the signal acquisition performance of GPS system that is currently in operation. In Section 4.6, the complexity of the proposed system is compared with that of GPS.

Table 1. Satellite signal receiver parameter.

Parameter \ Satellite		S1	S2	S3	S4
Gain		1	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{2}}$
Delay(sample)		0	250	500	750
Pilot	N_p	1023	1023	1023	1023
	Root	1	1	1	1
Data	N_d	20460	20460	20460	20460
	Root	1	17	41	103

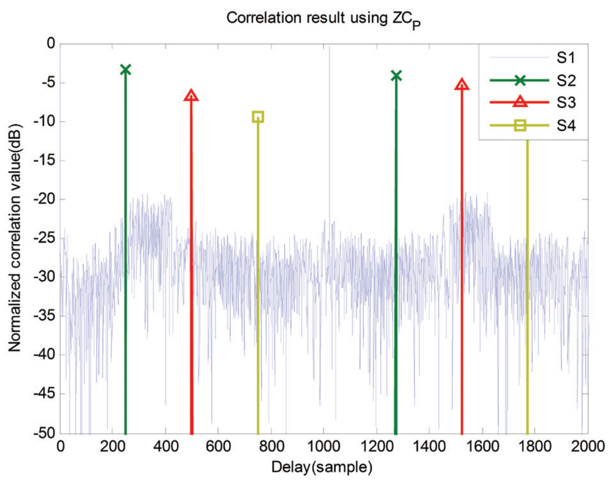

Fig. 6. Correlation with Zadoff-Chu sequence (Pilot).

4.1 Simulation environment and parameter

For testing the sign extraction through the signal scheme proposed in Chapter 2 and the correlation value mentioned in Chapter 3, four visible satellites, which are necessary for the navigation of actual receiver, are assumed. Table 1 shows the parameters which apply the receiving signal assumed in Chapter 3 to simulation based on Eq. (8).

The number of satellites (N_s) was 4, and the time delay τ_k for each satellite was made to maintain regular interval. Also, for applying the signal attenuation depending on time delay, the signal power was applied so that there is a difference of 3dB relative to S1. For the identical comparison in the future, the pilot signal of 1023 which is the C/A code length of GPS was used, and for the transfer rate of 50 bps, the data length of 1023 x 20 was used.

Fig. 5 shows the analysis of the mutual interference between pilot and data which follow the previously established parameters. When a satellite sends the identification information and data information as shown in Fig. 3, considerably small interference is observed though they have different lengths.

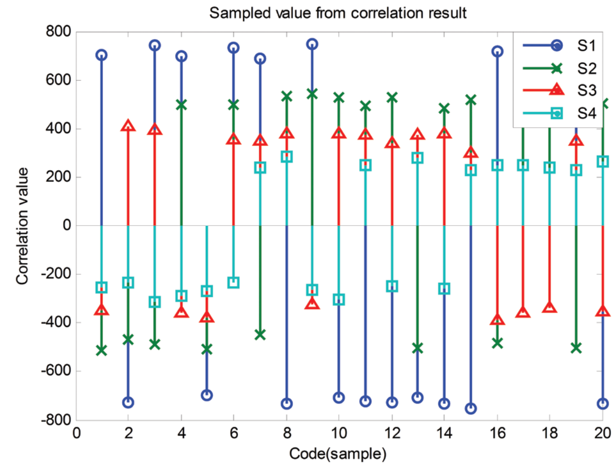

Fig. 7. Extracting PRN code from correlation using pilot signal.

4.2 Correlation with pilot signal to distinguish satellite

Fig. 6 is a diagram which magnified two periods of pilot signal among the correlation value results. Beginning with the S1 signal which first arrives at the receiver, the correlation value result for each satellite shows gradually decreasing peak value with the delay distance specified in Table 1. Also, the peak values of this normalized correlation value have the attenuation ratio of about 3 dB. The difference of 3 dB which was set in Table 1 is not precisely observed, which is thought to be the result of self-interference and differential interference. Though there is attenuation, the PRN code of satellite could be extracted for every sample of N_p if the threshold value is exceeded.

Fig. 7 only shows the real number value of the sample which corresponds to peak value from the result of Fig. 6. The PRN code assigned to each satellite can be identified. Similar to the previous result, the correlation value differs depending on the attenuation ratio which was initially set, and the effect of differential interference is also observed. The PRN code of every satellite, from which the signal is currently received, could be obtained through a single correlation value calculation that corresponds to the length of N_p.

Fig. 8 shows the correlation value between the PRN code extracted from the correlation value of receiving signal for searching the assigned satellite and the PRN code generated inside the receiver. It is found that the result follows the auto-correlation characteristic of PRN code. The PRN code is obtained for every ZC code period based on the peak value shown in Fig. 6. Thus, if the time delay difference of even a single chip exists between satellite signals, the advantage is that there exists no cross-correlation interference with the PRN code assigned to another satellite.

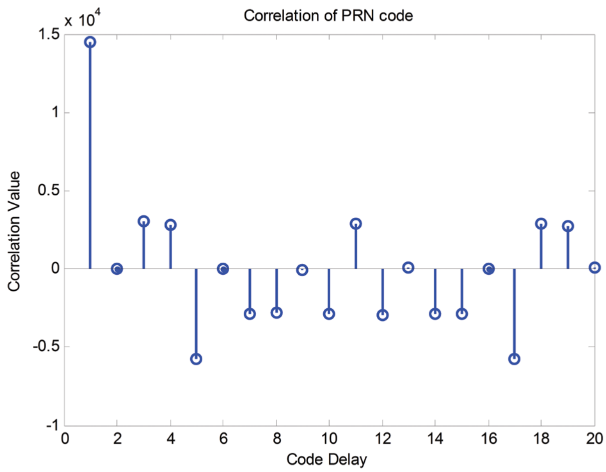

Fig. 8. PRN code correlation.

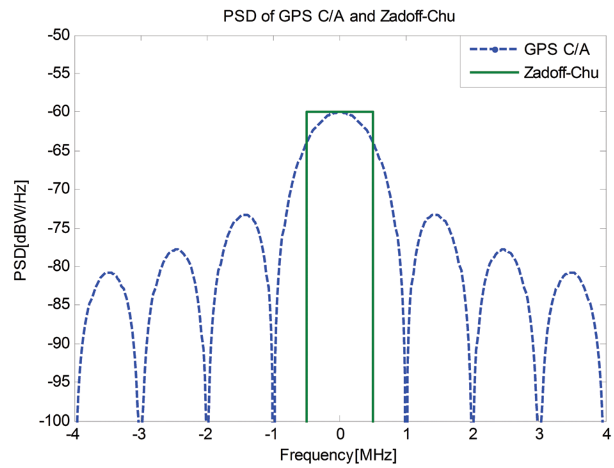

Fig. 10. PSD of CA and Zadoff-Chu code.

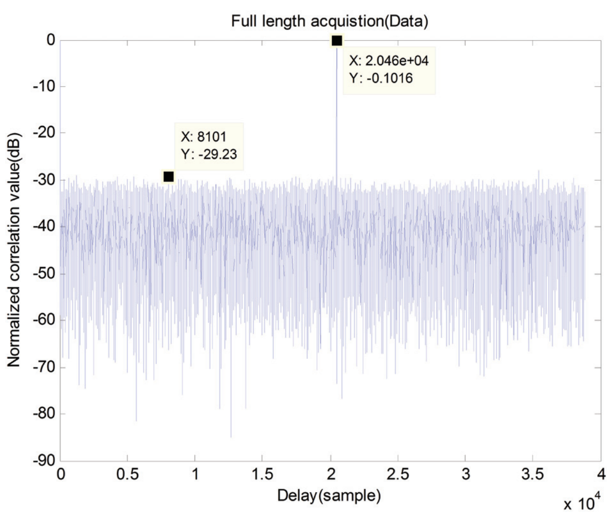

Fig. 9. Correlation for data acquisition.

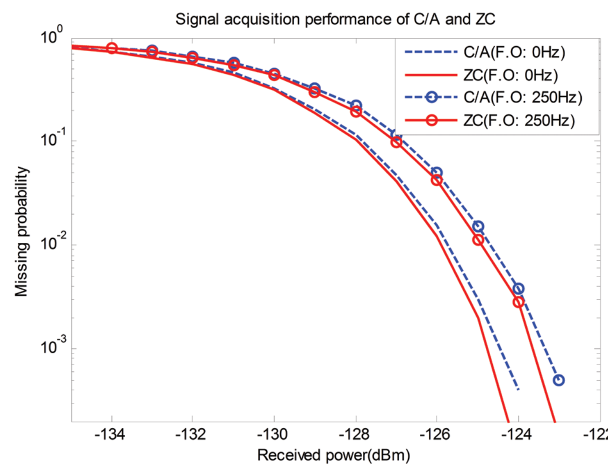

Fig. 11. Signal acquisition performance.

4.3 Correlation with data signal

The correlation value was taken as full length for receiving the data transmitted from each satellite, and the result is shown in Fig. 9. When acquiring the data, the interference from the pilot signal is about -29 dB, which is very small. After the signal acquisition, a stable signal tracking seems to be possible if the frequency synchronization and time synchronization are to some degree stabilized.

4.4 Power spectral density of C/A and Zadoff-Chu code

The C/A code has a sinc function shape in the frequency domain because it is the modulation system of Binary-Phase-Shift-Keying, and the ZC code has a constant value in the time and frequency domains as shown in Fig. 10. The bandwidth of main lobe for the C/A code is 2 MHz, while the ZC code has the main lobe of 1MHz. Thus, when the noise power to bandwidth is identical, the effect of noise for the ZC code is expected to be half of the effect of noise for the C/A code.

4.5 Acquisition performance under noise and Doppler shift environment

To consider the actual operation environment, in the environment with noise and Doppler shift, the code acquisition performances for the C/A code of GPS that is currently in operation and the proposed ZC code were analyzed at the same threshold value based on 1023 chips, and the result is shown in Fig. 11. Also, it was assumed that 10 satellite signals of different phases are received as the same gain. In Fig. 10, the ZC code was expected to have 3dB less noise effect than the C/A code, but the actual

Table 2. Comparing amount of calculation.

	GPS	Proposed System
Multiplication	$4 \cdot 21 \cdot 2046 \cdot 1023 \cdot Ns =$ 175816872·Ns	$4 \cdot (21 \cdot 2046 \cdot 1023 + 20^2 \cdot Ns) =$ 175816872 + 1600·Ns
Summation	$2 \cdot 21 \cdot 2046 \cdot 1023 \cdot Ns =$ 87908436·Ns	$2 \cdot (21 \cdot 2046 \cdot 1023 + 20^2) =$ 87908436 + 800·Ns

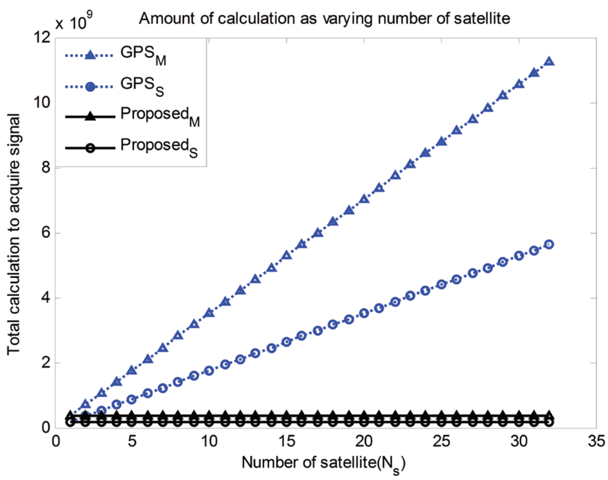

Fig. 12. Total calculation to acquire signal.

performance difference is insignificant because it is the signal scheme which combines the pilot signal that is 3 dB less and the data signal. However, regarding the interference between the satellite codes, the ZC code has much less interference than the C/A code. Thus, the proposed signal scheme has the gain of about 0.1 dB. Also, regarding the Doppler shift environment, the two codes showed the performance degradation of about 1dB when the Doppler frequency shift is 250 Hz. From the above results, it is shown that the performance of ZC code-based transmitting signal is not inferior to that of C/A code-based signal with respect to noise and Doppler frequency, and that more visible satellites are advantageous in term of code interference.

4.6 Complexity analysis

In the previous sections, the applicability of signal scheme was investigated which was proposed by analyzing the code acquisition for distinguishing satellites from the signal scheme modeled in Chapter 2, the data acquisition for obtaining the satellite navigation information, and the code acquisition performance under noise and Doppler shift environment. In this section, the improvement in the amount of calculation is quantitatively analyzed which can be achieved by the proposed signal scheme when acquiring the data. For this purpose, the number of cells at the code acquisition space is compared between the GPS system that is currently in operation and the proposed system.

For the Doppler frequency axis, if it searches the 5 kHz range by the 500 Hz, total 21 intervals are generated. For the code delay axis, if it searches by the half chip for preventing the loss of correlation value, 2,046 intervals are generated, and for the satellite code axis, if it has the intervals as many as the number of satellites (N_s), the GPS has the number of cells of N_{cell} = 21 x 2,046 x N_s (Petovello 2011). The proposed system does not have the search for satellite code axis, and only has the number of cells of N_{cell} = 21 x 2046. Each cell has 1023 complex number multiplications and 1022 complex number summations. When every cell is searched for acquiring the signal, the amount of calculation needed is shown in Table 2 and Fig. 12.

In the case of the proposed system, the calculation of PRN code correlation value for distinguishing satellites is added, but the amount of calculation is insignificant when compared to the GPS. As the dependence on the number of satellites is markedly different, the proposed system will be more efficient when the number of satellites to be searched is large.

5. CONCLUSION

In this paper, a signal scheme was described which could simplify the cell search of satellite code axis for reducing the complexity when acquiring the satellite signal, and the actual receiving signal was modeled. Using the correlation characteristic of ZC code, by adding the pilot signal which performed the mapping of PRN code for the existing data signal, the PRN code of every satellite, from which the signal is currently received, could be obtained through a single correlation value calculation. The data signal acquisition was also found to be achievable, and the advantage in the code interference was demonstrated by the analysis of code acquisition performance considering the actual environment. Through this, it was verified whether the proposed signal scheme could have the performance of a typical GNSS.

For a proper complexity comparison, the parameters which are identical to the GPS system were established, and the amount of calculation was compared and analyzed. As a result, the proposed satellite navigation system could lower the dependence of complexity depending on number of satellites, from the number of cells that the Doppler frequency and code delay axis comprise to the PRN code length, and the improvement of complexity was presented as table and figure.

However, when a signal, without time delay for each satellite signal where the synchronization matches exactly, is received, there could be an interference between the PRN codes. Therefore, the characteristic of PRN code, which is the secondary code for distinguishing satellites, should also be considered in the future.

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