Performance Analysis of Low-Order Surface Methods for Compact Network RTK: Case Study

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ABSTRACT

Compact Network Real-Time Kinematic (RTK) is a method that combines compact RTK and network RTK, and it can effectively reduce the time and spatial de-correlation errors. A network RTK user receives multiple correction information generated from reference stations that constitute a network, calculates correction information that is appropriate for one's own position through a proper combination method, and uses the information for the estimation of the position. This combination method is classified depending on the method for modeling the GPS error elements included in correction information, and the user position accuracy is affected by the accuracy of this modeling. Among the GPS error elements included in correction information, tropospheric delay is generally eliminated using a tropospheric model, and a combination method is then applied. In the case of a tropospheric model, the estimation accuracy varies depending on the meteorological condition, and thus eliminating the tropospheric delay of correction information using a tropospheric model is limited to a certain extent. In this study, correction information modeling accuracy performances were compared focusing on the Low-Order Surface Model (LSM), which models the GPS error elements included in correction information using a low-order surface, and a modified LSM method that considers tropospheric delay characteristics depending on altitude. Both of the two methods model GPS error elements in relation to altitude, but the second method reflects the characteristics of actual tropospheric delay depending on altitude. In this study, the final residual errors of user measurements were compared and analyzed using the correction information generated by the various methods mentioned above. For the performance comparison and analysis, various GPS actual measurement data were collected. The results indicated that the modified LSM method that considers actual tropospheric characteristics showed improved performance in terms of user measurement residual error and position domain residual error.

Keywords: network RTK, RTK, low-order surface methods, tropospheric delay

1. INTRODUCTION

Network Real-Time Kinematic (RTK) has been developed since the late 1990s to overcome the short coverage of existing RTK. Thus, when network RTK is used as infrastructure, users in a wide area can obtain precise position accuracy at a relatively cheaper establishment cost compared to RTK.

A network RTK user generates correction information that is appropriate for one's own position using the

Received Jan 29, 2015 Revised Mar 01, 2015 Accepted Mar 02, 2015 [†]Corresponding Author E-mail: kee@snu.ac.kr Tel: +82-2-880-1912 Fax: +82-2-888-2069 correction information generated by multiple reference stations that constitute a network based on a proper correction information combination method. Each correction information combination method performs the function of modeling the GPS error elements included in correction information in relation to space. A user calculates position by applying the combined correction information to one's own GPS measurements. When a network reference station modeled by each correction information combination method is closer to the actual GPS error elements of user position, the user can obtain more accurate position accuracy. Therefore, to effectively eliminate the GPS error elements of user position, the error element modeling accuracy of a correction information combination method is important. For this purpose, various correction information combination methods have been developed by many researchers. The simplest method is the Distance Interpolation Method (Gao et al. 1997), which models GPS error elements in inverse proportion to the distance between reference stations. Representative methods include Linear Interpolation Method (Schaer et al. 1999, Wanninger 1995) and Low-Order Surface Model (LSM) (Varner 2000), which perform modeling in relation to the horizontal or horizontal and altitude directions. Also, various multiple reference station correction information combination methods have been summarized and introduced by Dai et al. (2003) and Fotopoulos & Cannon (2001).

On the other hand, in the case of a method that models GPS error elements in relation to only the distance between reference stations or the horizontal direction among correction information combination methods, a tropospheric delay model is used to eliminate the tropospheric delay term among GPS error elements with a large effect from altitude. However, based on the analysis of the modeling error of the UNB3 tropospheric model using 10-year radiosonde data, Collins reported that the difference was more than 20 cm in 76 out of 1,011,651 data (Collins & Langley 1999). Therefore, to avoid an error from the use of a tropospheric model, the LSM method that considers GPS error element modeling in the altitude direction can be utilized. For the existing LSM method, a model equation can be obtained by the partial differentiation of a correction information equation with respect to the horizontal and vertical directions. On the other hand, based on the fact that refractivity, which is closely related with tropospheric delay, can be modeled as an exponential function in relation to altitude (Bean & Dutton 1966), a correction information combination method where the characteristics of the change in the tropospheric delay depending on altitude are combined with the LSM method that considers the altitude direction was suggested (Song et al. 2014).

In this study, the performances of the existing LSM method that considers the altitude direction and a modified LSM method that considers tropospheric delay characteristics depending on altitude were analyzed using various GPS data. First, based on the Continuously Operating Reference Stations (CORS) network located in mountainous terrain in the United States, the correction information combination performances were compared and analyzed using the GPS actual measurement data obtained under different climatic conditions. Also, based on the reference stations of the National Geographic Information Institute, Seoul Metropolitan Government and Seoul National University in Korea, performance verification was

also conducted in domestic environments. In Chapter 2, the existing LSM method and the modified LSM method that considers tropospheric delay characteristics were introduced. In Chapter 3, performance verification results using various GPS actual measurement data were presented, and they were compared and analyzed.

2. INTRODUCTION OF THE NETWORK RTK CORRECTION INFORMATION COMBINATION METHOD

2.1 Existing Low-Order Surface Correction Information Modeling Method

Low-order surface model methods have been suggested by a number of researchers (Wanninger 1995, Schaer et al. 1999, Varner 2000), and the correction information at an arbitrary position can be represented by master reference station correction information and its partial derivative value (Varner 2000). When the differenced correction information between reference stations is expressed as V, it can be classified into various cases depending on the order of the considered partial differentiation, as follows.

$$V = a + b \cdot \Delta x + c \cdot \Delta y$$

$$V = a + b \cdot \Delta x + c \cdot \Delta y + d \cdot \Delta z$$

$$V = a + b \cdot \Delta x + c \cdot \Delta y + d \cdot \Delta z + e \cdot \Delta z^{2}$$
(1)

In Eq. (1), Δx , Δy , and Δz represent the coordinate differences in the *x*, *y*, and *z* directions (local coordinate system), respectively, between a reference station and a specific position. Also, *a* is the constant term, *b*, *c*, and *d* are the first-order partial derivative terms of correction information with respect to the *x*, *y*, and *z* axes, respectively, and *e* is the second-order partial derivative term with respect to the *z* axis. Each variable is generally estimated based on the Least Square Estimation using the correction information equations of a number of reference stations. In this study, the *x*, *y*, and *z* coordinates were regarded as the local coordinate system, and they were defined as the east-west, south - north, and altitude directions, respectively.

2.2 Low-Order Surface Correction Information Modeling Method Considering Tropospheric Delay Characteristics

In this section, a modified LSM method suggested by our research team in 2014 by combining the existing LSM method with the characteristics between the troposphere and the altitude was introduced. Bean and Dutton reported



Fig. 1. (a) Selected reference station network in NC, US (b) Sky plot (US Dry) (c) Sky Plot (US Wet).

that refractivity, which is closely related with tropospheric delay, can be modeled as an exponential function in relation to altitude (Bean & Dutton 1966). Based on this, vertical tropospheric delay can be modeled as an exponential function depending on altitude; and using the Taylor series of the exponential function, the differenced vertical tropospheric delay equation between reference stations can be expressed as follows (Song et al. 2014).

$$T_{z}(z) = T_{z,0} \cdot e^{-kz}$$

$$\Delta T_{z} \equiv T_{z}(z_{2}) - T_{z}(z_{1}) = T_{z}(z_{1}) \cdot \left(-k\Delta z + \left(k\Delta z\right)^{2} - \cdots\right)$$
(2)

In the above equation, *z* represents the altitude. When up to the second-order term of the Taylor series are considered and it is expressed as an equation for slant using a mapping function and is then applied to the last equation in Eq. (2), the following constraints relevant to the modified LSM equation can be finally drawn. The constraint for the altitude difference when up to the second-order term of the Taylor series are considered is introduced in the reference (Song et al. 2014).

$$V = a + b \cdot \Delta x + c \cdot \Delta y + T_z(z_1) \cdot m(z_2) \cdot \left\{ -d \cdot \Delta z + e \cdot \Delta z^2 \right\}$$

where, $d = k > 0, e = d^2$ (3)

In Eq. (3), m represents the mapping function. Based on the fact that a tropospheric delay value decreases as the altitude increases, the first constraint of Eq. (3) (the variable d is larger than 0) can be drawn. Also, based on the relationship of the altitude first-order term and secondorder term coefficients observed in Eq. (2), a constraint that the variable e equals to the square of d can be obtained. For the mapping function in this study, the mapping function suggested by Black was used (Black & Eisner 1984).

3. EXPERIMENT RESULTS

3.1 Experiment Environment: CORS Reference Station Network in The United States (US Dry And Wet)

For the performance verification, GPS measurements of NCBR, NCMA, NCHE, MARI, and NCS, which are CORS reference stations in North Carolina, the United States, were collected between 13:00-15:00 EST, February 28, 2014 and between 08:00-11:00 EST, March 23. The NCBR reference station was used as the master reference station, and the NCSW reference station that is located at the center part of the network was set to be a static user as shown in Fig. 1a. Figs. 1b and 1c shows the sky plots at reference station. For the measurement collection on February 28, the relative humidity was less than 30%; and for the collection on March 23, the relative humidity was more than 90% due to rain. Measurements were collected at one second intervals, and the altitudes of the aforementioned reference stations are 747.4 m, 559 m, 643.1 m, 365.6 m, and 658.5 m, respectively. The maximum altitude difference was observed between the NCBR and MARI reference stations, which was approximately 400 m. On the other hand, for the coordinates of each reference station, the notified coordinates provided by the National Geodetic Survey in the United States, where phase center offset had been corrected, were used, and both the horizontal and vertical RMS errors of the notified coordinates were approximately 1 mm.

3.2 Experiment Environment: National Geographic Information Institute, Seoul Metropolitan Government, and Seoul National University Reference Station Network in Korea (KOR)



Fig. 2. Selected reference station network in South Korea and sky plot seen at master station.

To examine the performance of various correction information combination methods in Korea, the Paju (PAJU), Suwon (SUWN), and Dongducheon (DOND) reference stations of the National Geographic Information Institute, the Dobong (DBON) reference station established by Seoul Metropolitan Government, and the reference station installed at Building 301, Seoul National University (RS31) were used. The Paju reference station was used as the master reference station, and the Incheon, Seoul National University, and Dongducheon reference stations were used as the auxiliary reference stations, which established a single network as shown in Fig. 2. The Dobong reference station located within the network was selected to be a user, and GPS measurements were collected at one second intervals between 15:00-18:00 KST, January 8, 2015. The altitudes of the Paju, Suwon, Dongducheon, Seoul National University, and Dobong reference stations were 73.5 m, 83.8 m, 140.5 m, 281.9 m, and 73.1 m, respectively; and the maximum altitude difference was observed between the Seoul National University and Dobong reference stations (approximately 200 m). On the other hand, in the case of the reference station coordinates, the notified coordinates from the National Geographic Information Institute were used for the Paju, Suwon, and Dongducheon reference stations, and the notified coordinates from Seoul were used for the Dobong reference station. For the accuracy of the notified coordinates from the National Geographic Information Institute in 2014, the horizontal RMS error was ±5 mm + 0.5 PPM, and the vertical error was $\pm 5 \text{ mm} + 1 \text{ PPM}$. In the case of RS31, coordinate results with horizontal and vertical RMS of less than 1 mm that were obtained from Trimble Business Center, which is a precision post-processing program (Trimble), using 24-hour measurements were used. Also, during the processing of the reference station data, phase center offset was considered.

Master-Auxiliary Correction based (Euler et al. 2001) network RTK correction information was generated (Park & Kee 2010); and for the performance comparison, existing LSM that considers only the horizontal direction, the LSM method that considers horizontal direction first-order term and altitude direction second-order term, and a modified LSM method were used. The performance comparison was conducted by comparing and evaluating the correction information at the user position and the residual RMS error of the differenced user measurement values that were calculated by each correction information combination method. Also, final position domain residual errors for the user due to the residual errors were compared and analyzed. For both the US and KOR data, in the case of LSM without considering altitude, the troposphere was eliminated using the Black model (Black & Eisner 1984).

3.3 Experiment Results

To analyze accurate performance of the correction information combination methods, the distance term for user measurement was eliminated based on the precise position of the user. For this calculated user residual error, the correction information at the user position estimated by the various correction information combination methods is applied, and satellite differencing is then performed, which gives the residual error of the user. For this residual error, the effects of distance, receiver clock, and satellite clock are completely eliminated, and it only includes the double differenced values of the ionosphere, troposphere, satellite orbit error, and integer ambiguity. In this regard,

RMS error [m]	LSM	LSM altitude second-order	Modified LSM altitude second-order
US Dry	0.0098	0.0122	0.0099
US Wet	0.0136	0.0145	0.0123
KOR	0.0122	0.0278	0.0106

the integer ambiguity can be eliminated by estimating it using an integer ambiguity estimation algorithm. The final residual error where the estimated integer ambiguity has been eliminated generally acts as an error when a user with inaccurate position information finally calculates the position. In this study, distance was eliminated assuming that precise position of the user is known, and thus only the effect of the above residual error could be examined. For the performance analysis, the RMS values of the final user residual errors were summarized in Table 1.

In the case of the LSM that considers altitude secondorder term, the number of equations was smaller than that of unknowns to be estimated, and thus a minimum-

norm estimation method was used (Song et al. 2014). Also, in the case of the modified LSM method that considers tropospheric characteristics and altitude second-order term, nonlinear constraints were included, and thus unknowns were estimated using a parameter optimization technique. The results indicated that for the US Dry data with a low humidity, the RMS error values of the existing LSM method and the modified LSM method were almost similar, but the RMS error of the modified LSM method was approximately 20% smaller than that of the existing method that considers altitude second-order term. For the US Wet data with a high humidity, the modified LSM method showed 10% and 15% superior performance compared to the existing LSM method and the LSM method that considers altitude second-order term, respectively. For the KOR data collected in Korea, the modified LSM altitude second-order method showed 13% and 62% superior performance compared to the existing LSM method and the LSM altitude second-order method, respectively. It is thought that the LSM method



Fig. 3. L1 User residual errors for KOR data set (a) LSM, (b) LSM altitude second-order, (c) Modified LSM altitude second-order.

that considers altitude second-order term had larger RMS errors than the LSM method because the number of unknowns was smaller than that of measurements although the altitude term was considered. Fig. 3 shows the final L1 measurement user residual errors for the KOR data when the LSM, the LSM considering altitude second-order term, and the modified LSM were applied. In the figure, the red thick horizontal line represents the half wavelength of the L1 frequency.

The LSM method that considers altitude second-order term had a substantially higher noise level of residual error than the other two cases. This is thought to be because the condition of the matrix that is used for the estimation of unknowns for the LSM method considering altitude secondorder term and consists of the horizontal and vertical coordinate differences between master-auxiliary reference stations deteriorated compared to the LSM method. The modified LSM method used parameter optimization, and thus it is thought that the accuracy of the estimation of unknowns would not be solely determined by the geometric arrangement of the reference stations mentioned above.

On the other hand, the final user residual error acts as a position error in the calculation of the user position. To examine the effect, the residual error in the measurement domain can be converted to position domain errors in the east, north, and up directions using an observation matrix that consists of user line-of-sight vector. Fig. 4 shows the results.

Table 2 shows the horizontal and vertical RMS errors of

Table 2. User position errors for various low-order surface methods.

RMS error [m]		LSM	LSM altitude second-order	Modified LSM altitude second-order
US Dry	Horizontal error (2DRMS)	0.0234	0.0250	0.0239
	Vertical error (2RMS)	0.0401	0.0524	0.0341
US Wet	Horizontal error (2DRMS)	0.0367	0.0375	0.0336
	Vertical error (2RMS)	0.0687	0.0678	0.0652
KOR	Horizontal error (2DRMS)	0.0237	0.0547	0.0218
	Vertical error (2RMS)	0.0537	0.1154	0.0482

Table 3. Height differences between reference stations and master station for two test networks.

US network	NCMA-NCBR	NCHE-NCBR	MARI-NCBR
Altitude difference	188.4	94.3	381.8
KOR network	INCH-PAJU	RS31-PAJU	DOND-PAJU
Altitude difference	14.9	208.4	66.9

various correction combination methods for all test cases. For the data collected in the United States, the performance improvement of the modified LSM method was distinct on the rainy day with a high relative humidity. For the horizontal 2DRMS and vertical 2RMS errors on the day with a low relative humidity (US Dry), the existing LSM that considers altitude second-order term showed the lowest accuracy. In the case of the existing LSM and the modified LSM, the horizontal accuracy performance of the modified LSM decreased by approximately 2%, but the vertical accuracy increased by approximately 15%. For the rainy day (US Wet), the horizontal 2DRMS and vertical 2RMS errors of the modified LSM were improved by approximately 8% and 5%, respectively, compared to those of the existing LSM. In this case, the LSM considering altitude secondorder term also showed the lowest performance. On the other hand, when the KOR data were used, the existing LSM considering altitude second-order term also showed the lowest accuracy; and for both the horizontal and vertical errors, the accuracies of the modified LSM were improved by approximately 7% and 10% compared to those of the LSM, respectively.

The processing of the GPS measurements collected in the United States and Korea indicated that when the KOR data were used, the vertical positions of all the correction information combination methods were biased by approximately 2 cm compared to when the US data were used, as shown in Fig. 4c. These vertical bias errors could be due to the accuracy of the altitude direction coordinates for the reference stations in Korea. On the other hand, in the case of the existing LSM that considers altitude secondorder term, the horizontal and vertical accuracies using the KOR data were lower than those using the US data. For the corresponding method, a solution is calculated using the minimum-norm method when the number of measurements is smaller than that of unknowns, and the condition of the matrix that considers the horizontal and vertical geometry of reference stations has a large effect on the model parameter estimation accuracy. In the case of the US data, measurements were collected from the reference stations with a large altitude difference between reference stations as summarized in Table 3, and thus it is thought that the condition of the corresponding matrix was more advantageous. On the other hand, in the case of the modified LSM where the relationship between the troposphere and the altitude was considered for all the data, the horizontal and vertical RMS errors were improved by approximately 3 mm and 6 mm, respectively, compared to those of the existing LSM that considers simple altitude second-order term, as summarized in Table 2.

















(c)

Fig. 4. Horizontal and vertical position domain errors. (a) US Dry, (b) US Wet, (c) KOR

4. CONCLUSIONS

In this study, the performances of various correction information combination methods that determine network RTK correction information performance were verified and compared. A modified LSM method where the tropospheric delay characteristics relevant to altitude had been reflected in the existing LSM method that considers altitude term was introduced, and the performances of the suggested method and the existing method were compared and analyzed using the GPS measurements collected in the United States under two different climatic conditions and the measurements collected in Korea.

The results indicated that the modified LSM showed improved horizontal and vertical user accuracies compared to the existing LSM and the LSM considering altitude second-order term in most cases. In particular, for the measurements in the United States, the degree of user performance improvement was higher on the day with a high humidity than on the day with a low humidity. For the day with a low humidity, tropospheric delay could be substantially accurately estimated using a tropospheric model. Thus, the existing LSM and the modified LSM showed similar horizontal accuracies, but the modified LSM showed a higher vertical accuracy. It is thought that the LSM that considers altitude second-order term had the largest overall user residual errors because the model constant could not be accurately estimated since the number of reference stations was not sufficient. On the other hand, it is thought that for the day with a high humidity, the performance of the modified LSM was improved compared to the existing LSM because the accuracy of the tropospheric model deteriorated relatively. When the data in Korea were used, the modified LSM also showed the highest accuracy. It is thought that the modified LSM was generally effective for improving the horizontal and vertical user position accuracies compared to the other two methods although the degree was not large.

It is expected that the modified LSM that considers tropospheric delay characteristics would be effective for improving user position accuracy when the accuracy of a tropospheric model is not guaranteed as in a day with a high humidity.

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