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Analysis of GNSS Signal Acquisition Performance Spreading Zadoff-Chu Codes

Gwang Hee Jo[1], Yun Sub Choi[2], Deok Won Lim[3], Sang Jeong Lee[1†]

[1]Department of Electronics Engineering, Chungnam National University, Daejeon 34134, Korea
[2]FidusGEN, Daejeon 34027, Korea
[3]Satellite Navigation Team, KARI, Daejeon 34133, Korea

ABSTRACT

This paper analyzes the signal acquisition performance of the legacy GNSS spreading codes and a polyphase code. The code length and chip rate of a polyphase code are assumed to be same as those of the GPS L1 C/A and Galileo E1C codes. The autocorrelation and cross correlation characteristics are analyzed. In addition, a way to calculate a more accurate probability of false alarm for a code with sidelobe non-zero auto-correlation function is proposed. Finally, we estimate the probability of detection and the mean acquisition time for a given signal strength and the probability of false alarm.

Keywords: signal acquisition, PN sequence, Zadoff-Chu sequence

1. INTRODUCTION

In spreading spectrum communication, pseudo-noise sequence is used in order to use a wider band than the one required by the original signal. Code Division Multiple Access in communication systems can be performed through auto-correlation and cross-correlation characteristics of the pseudo-noise sequence. In the spreading spectrum receiver, de-spreading is performed with a replica sequence synchronized with the pseudo-noise sequence to recover the original signal.

The spreading codes used in communication systems can be classified into Pseudo Noise (PN) codes, Orthogonal codes, and Polyphase codes. PN codes are binary codes that have similar characteristics as random noise, and the most commonly used sequences are Maximal Length sequences, Gold sequences, and Kasami sequences (Holmes 2007). Orthogonal codes have zero cross-correlation characteristics, and the representative codes include Walsh codes and orthogonal gold codes. Polyphase codes are defined as complex roots of unity (Chu 1972). The Zadoff-Chu sequence is a typical polyphase code, which has Constant Amplitude Zero Auto-Correlation (CAZAC) characteristics (Han et al. 2017). Therefore, the Zadoff-Chu sequence can reduce cost and complexity of the power amplifier even more than the traditional Walsh code in the Universal Mobile Telecommunication System (UMTS). It is also used as Primary Synchronization Signal and Sounding Reference Signal in 3GPP LTE (Song & Shen 2010).

In GNSS, Gold sequences are used in GPS L1 and BeiDou B1, and Maximal Length sequences are used in GLONASS L1 (RISDE 2008). In addition, the use of secondary codes was considered in modern GPS or Galileo. Studies have also been performed on using polyphase codes as secondary codes (Han et al. 2017). Polyphase codes have better correlation characteristics than traditional PN codes or orthogonal codes, which is beneficial for detecting signals in GNSS.

In this paper, we analyzed GPS L1 C/A and Galileo E1 code and the signal acquisition performance using Zadoff-Chu codes as GNSS raging codes among various factors that

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†**Corresponding Author**
E-mail: eesjl@cnu.ac.kr
Tel: +82-42-825-3991 Fax: +82-42-823-5436

Gwang Hee Jo https://orcid.org/0000-0001-8732-9770
Yun Sub Choi https://orcid.org/0000-0002-9552-4469
Deok Won Lim https://orcid.org/0000-0002-5154-8063
Sang Jeong Lee https://orcid.org/0000-0002-9400-5157

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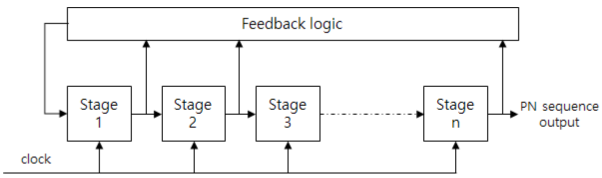

Fig. 1. PN sequence generator.

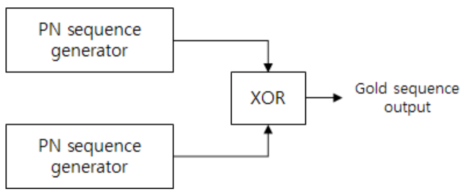

Fig. 2. Gold sequence generator.

influence the positioning performance. First, the code length was set to be the same as GPS L1 C/A and Galileo E1 code to analyze the autocorrelation characteristics. Then, we derived the probability of false alarm which is one of the signal acquisition performance indicators. At this time, we also proposed a method to calculate the probability of false alarm in a more accurate manner for codes in which the sidelobe of the correlation function is not 0, such as PN codes. Finally, this study predicted the probability of detection and the mean acquisition time from the given probability of false alarm for random signal strengths and compared their performances.

2. CORRELATION CHARACTERISTICS OF ZADOFF–CHU CODES

The PN codes have binary values and are generated similar to noise. The advantages of PN codes are that the codes are determined and they are easily generated by feedback shift register circuit. In addition, the autocorrelation function of this code has a high peak when the codes match, and is almost 0 when the codes do not match. The PN code generator has a structure as shown in Fig. 1, and the PN code generated by a N-stage shift register has a length of 2^N-1. An example of using PN code is GPS L1 C/A code (Dunn 2018).

The Gold sequence generator has a structure as shown in Fig. 2. The GPS L1 C/A code using the Gold sequence generator is generated by inputting the output of PN code generator into XOR gate. Similar to the PN code, GPS L1 C/A code is generated by a 10-stage shift register and has a length of 1023. The autocorrelation function of GPS L1 C/A code also produces a peak when the codes match, and a value close to 0 when the codes do not match. When the mainlobe is normalized to 1, the sidelobe has three values: 63/1023, -1/1023, and -65/1023. The autocorrelation results are shown

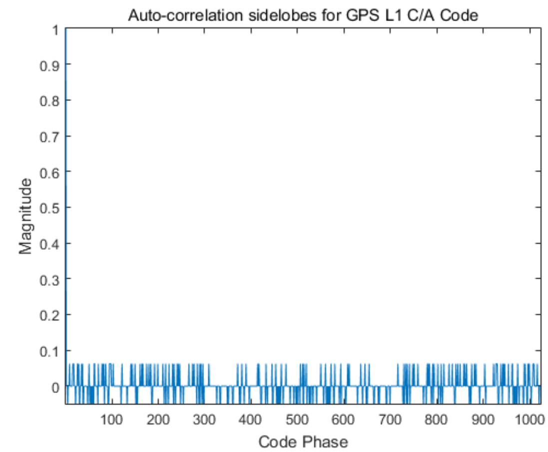

Fig. 3. Autocorrelation sidelobes for GPS L1 C / A code.

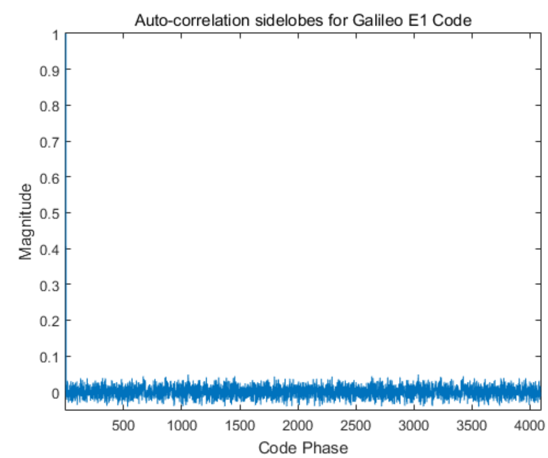

Fig. 4. Autocorrelation sidelobes for Galileo E1 code.

in Fig. 3 (Misra & Enge 2006). The maximum absolute value of the sidelobe is 0.0635.

The Galileo E1 primary code is a pseudo-random memory code sequence, which is 4092 in length, unlike GPS (GSA 2016). In terms of the autocorrelation results, when the mainlobe is normalized to 1, unlike the sidelobe of the GPS L1 C/A code with three values, it has various values. The autocorrelation results are shown in Fig. 4. The maximum absolute value of the sidelobe is 0.0479.

The Zadoff-Chu sequence implemented so that the polyphase code has CAZAC characteristics is as shown in Eq. (1).

$$C_{ZC}[n] = \begin{cases} e^{j\pi un(n+2q)/N_{ZC}}, & N_{ZC}\text{: even} \\ e^{j\pi un(n+1+2q)/N_{ZC}}, & N_{ZC}\text{: odd} \end{cases} \quad (1)$$

where N_{ZC} is the code length, u is an integer smaller than N_{ZC} but mutually exclusive, and q is a random integer. If the mainlobe is normalized to be 1 when the phases are matched, then the sidelobe is 0 when the phases are not matched

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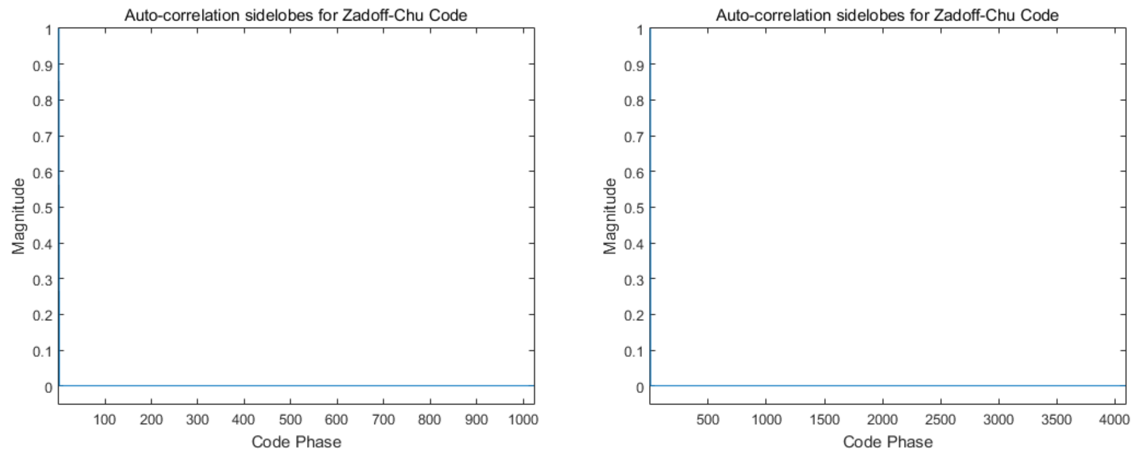

(a) (b)

Fig. 5. Autocorrelation sidelobes for Zadoff-Chu (a) 1023chips, (b) 4092chips.

(Chu 1972). If the code length N_{ZC} and the integer u are mutually exclusive, the CAZAC characteristics are obtained. The CAZAC characteristics by the Zadoff-Chu sequence are shown in Fig. 5.

Heimiller (1961) performed a research on the polyphase codes with good correlation characteristics. Good correlation characteristics refer to when the peak periodically appears if the phases of the codes match, while other correlation results are shown as 0. In terms of comparing the autocorrelation results of the normalized GPS L1 C/A code, Galileo E1 code, and the Zadoff-Chu sequence, the sidelobe size of Zadoff-Chu is close to 0, therefore, the Zadoff-Chu sequence has better correlation characteristics than GPS L1 C/A code and Galileo E1 code.

3. SIGNAL ACQUISITION PERFORMANCE

In signal acquisition theory, the probability to acquire signals when they exist can be defined as P_d in Eq. (2), and the probability to incorrectly acquire signals when they do not exist is as P_{fa} in Eq. (3), respectively.

$$P_d \quad \int^{\infty} p_s(z)dz,\ \text{p (z): pdf (presence of signal)} \qquad (2)$$

$$P_{fa} = \int_{V_t}^{\infty} p_n(z)dz,\ \text{p}_\text{n}\text{(z): pdf (signal absent)} \qquad (3)$$

where V_t is the threshold value to determine whether a signal is acquired. A signal is determined to exist when the envelope of the signal is greater than or equal to the threshold value. The envelope z of the signal is defined as $\sqrt{I^2+Q^2}$, and assuming that the Inphase/Quadrature correlation values of GPS receiver have respective Gaussian distributions, $\sqrt{I^2+Q^2}$, has a Ricean distribution when a signal exists. The Ricean distribution is as shown in Eq. (4) (Kaplan & Hegarty 2006).

$$p_s(z) = \begin{cases} \dfrac{z}{\sigma_n^2} e^{-\left(\frac{z^2+A^2}{2\sigma_n^2}\right)} I_0\left(\dfrac{zA}{\sigma_n^2}\right), & z \geq 0 \\ 0, & z < 0 \end{cases} \qquad (4)$$

where σ_n^2 is the RMS noise power, A is the RMS signal amplitude, and $I_0(zA/\sigma_n^2)$ is the zero-order modified Bessel function. In Eq. (4), when $(A=0)$ in case there is no signal, a Rayleigh distribution is obtained as shown in Eq. (5).

$$p_n(z) = \frac{z}{\sigma_n^2} e^{-\left(\frac{z^2}{2\sigma_n^2}\right)} \qquad (5)$$

Eq. (5) only considers $(A=0)$ when there is no signal, and does not consider when the sidelobe is not 0 as shown in Fig. 3. Considering GPS L1 C/A Code, for example, the effects of the three sidelobes of 63/1023, -1/1023, and -65/1023 were not considered when the Replica Code phase does not match. In order to calculate the probability of detection or false alarm considering the influence of the sidelobe, should also be assumed as a Ricean distribution. Then, the probability of false alarm can be calculated as shown in Eq. (6) by calculating the false alarm in the corresponding phase while shifting the code phase and obtaining the average over one period of the code.

$$P_{fa,sl} \triangleq \frac{1}{N-1} \sum_{i=1}^{N-1} \int_{V_t}^{\infty} \frac{z}{\sigma_n^2} e^{-\left(\frac{z^2+A_i^2}{2\sigma_n^2}\right)} I_0\left(\frac{zA_i}{\sigma_n^2}\right) dz,\ i = 1,2,3,\cdots \qquad (6)$$

where, i is an integer smaller than N, and N is the code chip sequence length. In this paper, N is 1023 or 4092 because GPS L1 C/A code, Galileo E1 code, and Zadoff-Chu code are compared. A_i is the correlation value when the phase of

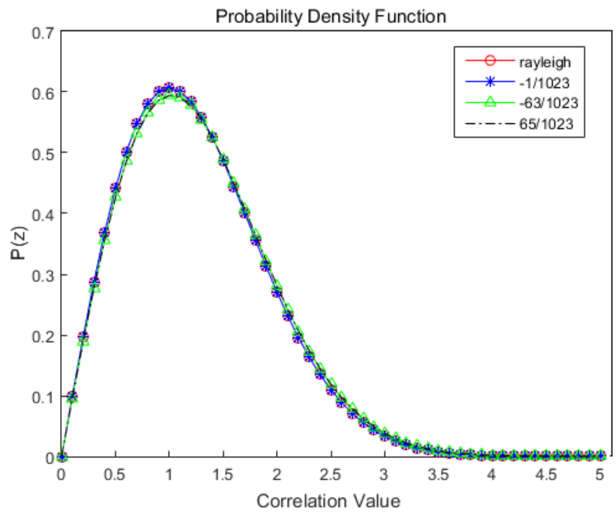

Fig. 6. Ricean, Rayleigh PDF.

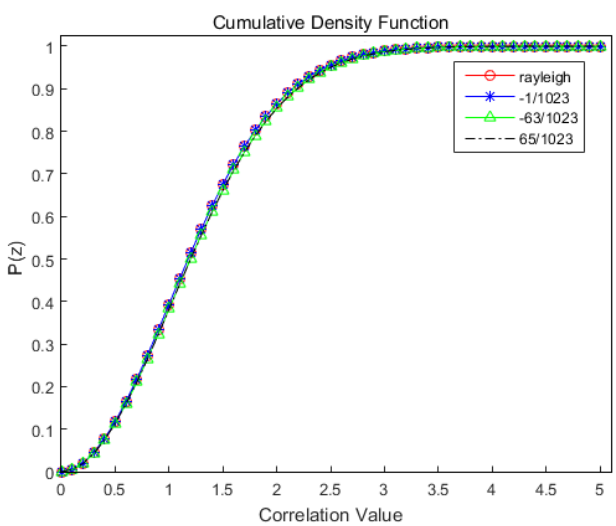

Fig. 7. Ricean, Rayleigh CDF.

the received code and the replica code differ by i chips. The sidelobe in the correlation value is not 0. When considering the influence of the sidelobe, it follows the Ricean distribution instead of the Rayleigh distribution because the signal power is present along with the noise. Figs. 6 and 7 show the probability density function (PDF) and cumulative distribution function (CDF) of the Ricean distribution according to the size of the sidelobe and the Rayleigh PDF and CDF assuming that there is only noise, when C/N0 is assumed to be 40 dB.

When PDF and CDF considering the sidelobe are used in order to obtain the probability of detection and false alarm, the probability of false alarm in signal acquisition theory changes. When the probability of false alarm and signal acquisition are given, the mean signal acquisition time is as shown in Eq. (7) (Park et al. 2002).

Table 1. Signal acquisition threshold according to code type (V).

Code type	Signal acquisition threshold
GPS L1 C/A	3.05019931122
Zadoff-Chu (1023)	3.03485425877
Galileo E1	3.03838091150
Zadoff-Chu (4092)	3.03485425870

Table 2. Average signal acquisition probability according to signal power (%).

C/N [dB]	GPS L1 C/A	Zadoff-Chu (1023)	Galileo E1	Zadoff-Chu (4092)
20	1.43	1.49	1.48	1.49
25	2.65	2.75	2.73	2.75
30	8.20	8.45	8.39	8.45
35	36.28	36.88	36.74	36.88
40	94.04	94.23	94.18	94.23

$$E\left[T_{acqu}\right]=(N-1)(T_d+T_{fa}P_{fa})\frac{(2-P_d)}{2P_d}+\frac{T_d}{P_d} \qquad (7)$$

where N is the maximum number of code phase bin to be searched, T_d is the dwell time, and T_{fa} is the dwell time caused by the false alarm. N is twice the code length because it searches the code phase at half-chip intervals. T_d uses GPS L1 C/A code period of 1 ms. T_{fa} is the product of the false alarm penalty (k_p) and the dwell time (T_d), in which the false alarm penalty is defined as 5.

This study compared the signal acquisition performance of the Zadoff-Chu code and the conventional GNSS spreading code by simulation. At this time, in Eq. (4), u uses 1 which is mutually exclusive with all integers, and q uses 0 which means phase shift. The carrier frequency and phase were assumed to be synchronized. In terms of calculating Eqs. (2) and (3), if the probability of false alarm is 1%, the probability of signal acquisition is 94.2235%. If the probability of false alarm becomes 0.1% by increasing the signal acquisition threshold, the probability of signal acquisition becomes 81.0823%. If the probability of false alarm is lowered, the probability of signal acquisition is also lowered. Therefore, Table 1 shows the signal acquisition thresholds at which the probability of false alarm is 1% when the signal power is 40 dB. As shown in Fig. 6, PDF moves to the right as size of the sidelobe increases. This increases the signal acquisition threshold that has a probability of false alarm of 1%. Since the Zadoff-Chu code has a smaller sidelobe size, the signal acquisition threshold is lower than GPS L1 C/A and Galileo E1.

Based on the signal acquisition thresholds in Table 1, the probability of detection and false alarm were calculated by changing the signal power from 20 dB to 40 dB. The PDF of the signal moves according to the signal power. The lower the signal power, the probability of signal acquisition of Eq. (2) becomes low. Table 2 shows the mean signal acquisition

Table 3. Average signal acquisition time according to signal power (s).

C/N [dB]	GPS L1 C/A	Zadoff-Chu (1023)	Galileo E1	Zadoff-Chu (4092)
20	11783	11284	45583	45137
25	6308	6069	24491	24278
30	1981	1921	7738	7684
35	382	374	1505	1499
40	95.4	95.1	380.8	380.5

probability according to the signal power. Table 3 shows the mean signal acquisition time from the given probability of false alarm and the derived probability of signal acquisition using Eq. (7).

4. CONCLUSION

This paper analyzed the correlation characteristics between GPS L1 C/A code and Galileo E1 code, which are the codes used in conventional satellite navigation systems, and the Zadoff-Chu code generated in the same length, and presented signal acquisition performance indicators to compare the performance. The correlation characteristics of the Zadoff-Chu code are better than GPS L1 C/A code and Galileo E1 code, so the correlation characteristics are closer to 0 except for the peak. As a result, the probability of false alarm due to the sidelobe was low and signal acquisition threshold was lowered. In addition, it was confirmed that the probability of signal acquisition and the acquisition time were improved according to the signal power. Therefore, the signal acquisition performance is expected to improve by using the Zadoff-Chu as GNSS ranging code. In future, we intend to compare the signal acquisition performance of 10,230 chips, which is the code length corresponding to GPS L1C and BeiDou B1C, and analyze the frequency ambiguity problems that occur when using the Zadoff-Chu code to derive a solution.

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AUTHOR CONTRIBUTIONS

The Manuscript with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used “conceptualization, G. H. Jo and Y. S. Choi; methodology, D.W.; software, G.H.; validation, G.H., Y.S. and D.W.; formal analysis, G.H.; investigation, G.H. and Y.S.; resources, G.H.; data curation, G.H.; writing --original draft preparation, G.H.; writing --review and editing, D.W.; visualization, G.H.; supervision, S.J.; project administration, Y.S.; funding acquisition, S.J.”. Authorship must be limited to those who have contributed substantially to the work reported.

CONFLICTS OF INTEREST

Authors must identify and declare any personal circumstances or interest that may be perceived as inappropriately influencing the representation or interpretation of reported research results. Declare conflicts of interest or state “The authors declare no conflict of interest.”

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Gwang Hee Jo received the B.S degrees in the Department of Electronics Engineering from Chungnam National University, Korea in 2017. His research interest includes GNSS receiver, digital signal processing.

Yun Sub Choi is currently working in FidusGen as a research engineer. He received the Doctor's degree in Electronics from Chungnam National University in 2017. His research interests include GNSS receivers and anti-jamming techniques.

Deok Won Lim received the B.S and Ph.D degrees in the Department of Electronics Engineering from Chungnam National Universi ty, Korea in 2004 and 2011, respectively. He is now working in Korea Aerospace Research Institute. His research interests include GNSS receiver design and anti-jamming technologies.

Sang Jeong Lee is currently a Professor in the Department of Electronics Engineering at Chungnam National University, Daejeon, Korea. He received the Doctor's degree in Control and Measurement in Seoul National University in 1987. His research interests include GNSS and Robust Control.