Positioning of Wireless Base Station using Location-Based RSRP Measurement

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ABSTRACT

In fingerprint-based wireless positioning, it is necessary to establish a DB of the unmeasured area. To this end, a method of estimating the position of a base station based on a signal propagation model, and a method of estimating the information of the received signal in the unmeasured area based on the estimated position of the base station have been investigating. The purpose of this paper is to estimate the position of the base station using the measured information and to analyze the performance of the positioning. Vehicles equipped with a GPS receiver and signal measuring equipment travel the service area and acquire location-based Reference Signal Received Power (RSRP) measurements. We propose a method of estimating the position of the base station using the measured information. And the performance of the proposed method is analyzed on a simulation basis. The simulation results confirm that the accuracy of the positioning is affected by the measured area and the Dilution of Precision (DOP), the accuracy of the position information obtained by the GPS receiver, and the errors of the signal included in the RSRP. Based on the results of this paper, we can expect that the position of the base station can be estimated and the DB of the unmeasured area can be constructed based on the estimated position of the base stations and the signal propagation model.

Keywords: RSRP, signal propagation model, location estimation

1. INTRODUCTION

In emergency rescue situations, the requester's location information is essential for successful rescue operation. If the requester is outdoors, his/her location can be easily obtained through the global positioning system (GPS) information received from the requester's terminal. However, if the requester is in the GPS shadow area, the location can be estimated by the wireless positioning technique through the mobile telecommunication / wireless communication infrastructure. A number of studies have been conducted

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Seong Yun Cho https://orcid.org/0000-0002-4284-2156 Chang Ho Kang https://orcid.org/0000-0002-9899-3076 on indoor and outdoor wireless positioning techniques. Among them, fingerprint method is known to be the most accurate technique. When using the fingerprint method, a fingerprint database (DB) in the service area should be set up in advance, and it requires much cost, manpower, and time to build and periodically update the DB (Li et al. 2005, Cho & Park 2014). In particular, the DB for emergency rescue is not yet established to cover the whole nation.

The development of basic fingerprint DB is needed to store signal information (in this study, reference signal received power (RSRP) is used) acquired through actual measurements along with the position information. The narrower the interval between measurements, the better the accuracy of location estimation based on it. Therefore, a method of estimating the signal information of the unmeasured positions by interpolation using the measured information was also studied (Li et al. 2005, Cho & Park 2014). However, if the unmeasured area is wide, it



Fig. 1. Service area.

is not possible to generate signal information of the area through interpolation. If the position of the base station and the propagation characteristics of the signal are known, the signal information of the unmeasured area can be estimated. A large number of studies have been conducted on signal propagation characteristics, and relatively reliable signal propagation models have been proposed (Hamid & Kostanic 2013, Zyoud et al. 2016, Liu et al. 2017). However, there is a fading problem due to non-line-of-sight (NLOS) and multi-path signals in urban areas, which makes model accuracy degraded. In addition, the position information of the base station is not disclosed due to security reasons of telecommunication service providers. Thus, a problem of estimating the position of the base station remains to create signal information of an unmeasured area based on signal propagation characteristics. Nonetheless, few studies have been conducted on this issue. Therefore, in this paper, we propse a method for estimating the location of a base station using measurement information. The performance of the proposed method is verified through simulations. The location-based RSRP measurements are acquired through GPS receivers and measurement equipment mounted in a vehicle, and the accuracy of location estimation of a base station according to the dilution of precision (DOP) and signal errors is analyzed.

2. POSITIONING OF WIRELESS BASE STATION

Consider the service area as shown in Fig. 1. The 112/119 vehicles mounted with a GPS receiver and wireless signal measuring instruments travel in the service area and collect measurements for the patrol and dispatch purposes. Unfortunately, the vehicles cannot visit all places in the service area to collect measurements. The locations of the

base stations are estimated using the location-based RSRP measurements. In this process, the signal propagation model and location estimation algorithm are used together. Once the position of the base station is estimated, measurements that can be acquired in the unmeasured area are virtually generated. The principle of virtual measurement generation is to calculate RSRP measurements based on the signal propagation model in the area receiving the signal transmitted from the base station whose location is estimated.

2.1 Wireless Signal Propagation Model

A path loss of the wireless signals transmitted from the base station is caused while they propagate through the air. As a result, the signal power weakens gradually, which in general can be modeled by a log function. The channel characteristics of wireless communication can also be analyzed by the NLOS signal due to structures such as buildings, multi-path fading, Doppler effect, etc. To analyze wireless channels accurately, ray tracing can be employed. To do this, however, accurate three-dimensional (3D) map, the materials of structures, their numerical characteristic information, and complex environment information are needed. Thus, the wireless channel characteristics can be analyzed using a simple path loss model, as presented in Eq. (1), in the wireless location estimation field (Zyoud et al. 2016).

$$PL = A \log_{10}\left(\frac{r}{r_0}\right) + B \log_{10}\left(\frac{f}{k}\right) + C + \beta r + qL_{wi} + X(\sigma)$$
 (1)

where, *A*, *B*, *C*, β , *q*, and σ are the design parameters that should be set according to the signal propagation environment, r_0 is the reference distance [m], *r* is the distance between base station and reception terminal [m], *f* is the signal frequency [GHz], and $X(\sigma)$ is the log-normal random number whose standard deviation is σ .

 β and q are used as parameters to consider additional attenuation occurring when signals penetrate walls. β is used to set a physical parameters such as wall penetration, building size, etc. and q means the number of separated walls (buildings) between base station and reception terminal. However, the above parameters can be used only when accurate environment information between base station and reception terminal is given. In addition, σ can be set differently according to LOS and NLOS environments and can also be interpreted as signal noise. Various path loss models according to design parameters have been studied, which can be categorized into 3GPP model, IEEE 802 model, WINNER II, etc. These models are derivatives of the radio frequency propagation model, and focus more on simulating realistic propagation environments, than existing models such as log-distance model, Cost 231-Hata model, Ericsson model, Lee model, etc.

Because environment information between base station and data measurement location cannot be known outdoors, a simplified path loss model as presented in Eq. (2) may be used.

$$PL = PL(r_0) + 10n \log_{10}(r) + X(\sigma)$$
(2)

where, n refers to a mean path loss index.

2.2 Wireless Positioning

The position of the base station is estimated by collecting location-based signals transmitted from the base station whose location is not known. The measurement equation can be represented as presented in Eq. (3).

$$\sqrt{dE_{B-R(i)}^2 + dN_{B-R(i)}^2 + dU_{B-R(i)}^2} = r(P_{R(i)})$$
(3)

where, $r(P_{R(i)})$ refers to the distance information calculated using the measured data $P_{R(i)}$ acquired at the reference location R(i), and $dx_{B-R(i)}$ refers to the distance information in the $x \in \{E, N, U\}$ axis between the base station and the reference location R(i), which can be calculated using Eq. (4).

$$dE_{B-R(i)} = (Lon_B - Lon_{R(i)})R_t \cos(Lat_{R(i)})$$
(4a)

$$dN_{B-R(i)} = (Lat_B - Lat_{R(i)})R_m$$
(4b)
$$dU_{B-R(i)} = H_B - H_{R(i)}$$
(4c)

where,
$$[Lat_B Lon_B H_B]^T$$
 and $[Lat_{R(i)} Lon_{R(i)} H_{R(i)}]^T$ refer to the location vectors of the base station and the reference location $R(i)$, respectively, which are expressed by latitude, longitude, and altitude. In addition, the Earth's radius in the latitude and longitude directions is calculated by Eq. (5) (Farrell & Barth 1999).

$$R_m = R_0 (1 - e^2) / \sqrt{(1 - e^2 \sin^2 Lat_{R(i)})^3}$$
(5a)

$$R_{t} = R_{0} / \sqrt{1 - e^{2} \sin^{2} Lat_{R(i)}}$$
(5b)

where, $R_0 = 6,378,137$ [*m*] and e = 0.0818191908 refer to the Earth's equatorial radius and eccentricity, respectively.

If Eq. (2) is expressed as signal propagation model, Eq. (6) can be obtained.

$$R(r) = R(r_0) - 10n \log_{10}(r) + X(\sigma)$$
(6)

where, R(r) refer to the RSRP acquired from the reception terminal, which is distanced *r* from the base station.

In Eq. (3), $r(P_{R(i)})$ can be calculated by Eq. (7), with the noise part in Eq. (6) ignored.

$$r = 10^{\alpha}$$
, where $\alpha = \frac{R(r_0) - R(r)}{10n}$ (7)

When measurements are acquired from M reference locations, assuming the position of the base station is $[Lat_B^* Lon_B^* H_B^*]^T$, which becomes the nominal point, linearization is performed, as presented in Eq. (8), through the first order Taylor series expansion of Eq. (3).

$$\frac{dE_{B-R(1)}^{*}}{r_{B-R(1)}^{*}} \quad \frac{dN_{B-R(1)}^{*}}{r_{B-R(1)}^{*}} \quad \frac{dU_{B-R(1)}^{*}}{r_{B-R(1)}^{*}} \\ \vdots \qquad \vdots \qquad \vdots \\ \frac{dE_{B-R(M)}^{*}}{r_{B-R(M)}^{*}} \quad \frac{dN_{B-R(M)}^{*}}{r_{B-R(M)}^{*}} \quad \frac{dU_{B-R(M)}^{*}}{r_{B-R(M)}^{*}} \end{bmatrix} \begin{bmatrix} dLon_{B} \\ dLat_{B} \\ dH_{B} \end{bmatrix} = \begin{bmatrix} r(P_{R(1)}) - r_{B-R(1)}^{*} \\ \vdots \\ r(P_{R(M)}) - r_{B-R(M)}^{*} \end{bmatrix}$$
(8)
$$\Leftrightarrow SX = R$$

where, $r_{B-R(i)}^* = \sqrt{dE_{B-R(i)}^{*2} + dN_{B-R(i)}^{*2} + dU_{B-R(i)}^{*2}}$ is a calculated value using the initial position information of the base station.

Thus, the location error with respect to the initial position of the base station can be estimated using the least square method, as presented in Eq. (9).

$$\hat{X} = (S^T S)^{-1} S^T R \tag{9}$$

The position of the base station is then compensated using the estimated location error. Then, this information is set to the initial position of the base station again, and then Eqs. (6-7) are performed iteratively to estimate the position of the base station (Mendel 1995).

2.3 Error Analysis

The location estimation technique of the base station, which is proposed in this study, includes various error factors. Their effects can be analyzed as follows: First, the errors are caused by the difference between signal propagation model and actual measurements, as mentioned above. It can be expressed as Eq. (10) by including the errors in Eq. (6).

$$R(r) = R(r_0) - 10n \log_{10}(r) + E(r) + X(\sigma)$$
(10)

where, E(r) represents all parameters (characteristics in relation to signal frequencies and obstacle penetration attenuation) ignored in Eq. (1) as signal errors. These errors are dependent on the location. However, since this information is unknown, it can be expressed as a function of

the distance between the base statoin and the receiver.

By arranging Eq. (10) with regard to $r(P_{R(i)})$, Eq. (11) can be obtained.

$$r(P_{R(i)}) = 10^{(R(r_0) - R(r_i))/10n} \cdot 10^{E(r_i)/10n} \cdot 10^{X(\sigma)/10n}$$
(11)

It can be expressed as Eq. (12) through the first order Maclaurin series expansion of Eq. (11).

$$\begin{aligned} r(P_{R(i)}) &\cong \overline{r}(P_{R(i)}) \left(1 + \frac{\ln 10}{10n} E(r_i)\right) \left(1 + \frac{\ln 10}{10n} X(\sigma)\right) \\ &\cong \overline{r}(P_{R(i)}) \left(1 + \frac{\ln 10}{10n} E(r_i) + \frac{\ln 10}{10n} X(\sigma)\right) \\ &= \overline{r}(P_{R(i)}) + \overline{r}(P_{R(i)}) \frac{\ln 10}{10n} E(r_i) + \overline{r}(P_{R(i)}) \frac{\ln 10}{10n} X(\sigma) \end{aligned}$$
(12)

where, $\overline{r}(P_{R(i)}) = 10^{(R(r_b)-R(r))/10n}$. Thus, the distance calculation error can be explained by Eq. (13).

$$\delta r(P_{R(i)}) = \overline{r}(P_{R(i)}) \frac{\ln 10}{10n} (E(r_i) + X(\sigma))$$
(13)

The location estimation error due to the distance calculation error can be expressed as Eq. (14).

$$\delta \hat{X} = (S^T S)^{-1} S^T [\delta r(P_{R(1)}) \cdots \delta r(P_{R(M)})]^T$$
$$= (S^T S)^{-1} S^T \frac{\ln 10}{10n} \begin{bmatrix} \overline{r}(P_{R(1)}) (E(r_1) + X(\sigma)) \\ \vdots \\ \overline{r}(P_{R(M)}) (E(r_M) + X(\sigma)) \end{bmatrix}$$
(14)

Eq. (14) implies that the location estimation error occurs due to the signal propagation model's error, signal noise, and DOP. DOP contains part of Eq. (14) and is calculated as in Eq. (15) (Kaplan 1996).

$$DOP = \sqrt{trace(S^T S)^{-1}}$$
(15)

That is, the smaller the DOP, the smaller the location estimation error. To this end, the measured locations should be evenly distributed around the base station. However, since the position of the base station is unknown, measurement data should be obtained in the overall service area as much as possible.

Another error factor is the measurement position error. A GPS receiver is used with the signal measurement terminal to store the measurement data and position information together. GPS-based position information may have a large error in urban areas. This error can affect Eq. (4). In addition, it also affect both of matrices S and R in Eq. (8). This tells that the location estimation error of the base station occurs due to this error. Because it is difficult to express this numerically, the location estimation error due to this error is analyzed



Fig. 2. Measurement acquisition trajectory.

based on simulations.

3. SIMULATION ANALYSIS

Simulations were conducted to analyze the performance of the proposed technique. The environment setup of simulation is described as follows:

- Mean path loss index n = 2.0 (Zyoud et al. 2016)
- *σ* = 1, 3, and 5 (dBm)
- $R(r_0) = -30.0 \, (\text{dBm})$
- GPS error (standard deviation) = 5, 30, and 100 (m)
- No. of Monte-Carlo simulations = 1,000
- Data acquisition cycle = 1 (Hz)

Fig. 2 shows the position of the base station and the trajectory for each step of acquiring the measurement. The measurement system is mounted in a vehicle, which runs around the trajectory. In Steps 1 to 3, data is measured while the vehicle runs in the same section iteratively. That is, the algorithm performance is analyzed when the signal measurement is obtained only one side around the base station, and the effect of iterative measurement in that section is analyzed. Steps 4 to 8 constitute a scenario that acquires data while the vehicle runs in new sections.

Fig. 3 shows the distance information calculated at each reference location and the position of the base station estimated in each step. Here, σ and GPS error were set to 3 dBm and 30 m, respectively. When the standard deviation of the signal error included in the RSRP was set to 3 dBm, a relatively large distance calculation error occurred. And the positioning error of the GPS whose standard deviation was 30 m considering the urban area was also deviated significantly from the actual trajectory. In contrast, the position of the base station was estimated near the true position, and the



Fig. 3. Positioning & Ranging results. (a) step 1 (b) step 2 (c) step 3 (d) step 4 (e) step 5 (f) step 6 (g) step 7 (h) step 8



Fig. 3. Continued





Fig. 3. Continued

standard deviation of the estimated error generated in 1,000 estimations was gradually reduced as the step progressed. The reason for the small error of the location estimation of the base station, compared to the large distance estimation error due to noise, was that because the noise was 0-mean normal distribution and the number of measurements is great, the position information of the base station estimated based on the least square method had a small error. The trend of the location estimation error according to σ , GPS errors and the step is analyzed as follows.

In Fig. 4, σ is set to 5 dBm, and the location estimation error according to GPS errors and trajectory in each step is displayed. As seen from this figure, the location estimation error according to the Monte-Carlo simulation was not significantly affected by the GPS error. That is, if errors are included in the GPS-based position information acquired

at the measurement environment, the errors did not significantly influence the location estimation of the base station. On the other hand, the trajectory step significantly affects the location estimation error of the base station. This phenomenon is analyzed through Fig. 2. The locations in Steps 2 to 3 are not different from that collected in Step 1. That is, this is the case of the collection vehicle iteratively running in the measured location. In this case, the mean of the location estimation error did not decrease through the additional measurement information. However, as the number of measurement data increased, the standard deviation of the location estimation error decreased. The location estimation accuracy was significantly improved because the measured locations were evenly distributed around the position of the base station while Steps 4 to 5 were performed. After this, once Steps 6 to 8 were performed,



Fig. 4. Position estimation error according to GPS error and step when σ is 5 dBm. (a) mean (b) standard deviation



Fig. 5. Position estimation error according to step and σ when GPS error is 5 m. (a) mean (b) standard deviation (c) DOP

in terms of the location estimation accuracy, errors were reduced as the number of measurements increased.

Fig. 5 shows the location estimation error according to σ and trajectory step when the standard deviation of GPS errors is set to 5 m. Overall, as the numbers of measurement locations and data increased, the location estimation error tended to decrease. However, the error reduction was large when σ was large, compared to when it was small. In the same step, the location estimation accuracy differs

significantly according to the size of σ . This is because the distance calculation error that occurrs when the distance is calculated through the RSRP measurements is largely reflected in the location estimation. DOP gradually decreased as the steps progressed. As a result, the accuracy is expected to improve as the steps progressed. However, in this case, the signal error included in the RSRP was more influential in location estimation than the accuracy of GPS-based position information and DOP. Table 1 presents the numerical

Index	Step	$\sigma = 1$ (dBm) GPS error			σ=3 (dBm) GPS error			σ= 5 (dBm) GPS error		
		5 m	30 m	100 m	5 m	30 m	100 m	5 m	30 m	100 m
Mean	1	16.582	16.502	18.307	97.724	95.229	90.079	269.734	267.971	258.600
	2	13.540	13.354	13.362	93.252	92.333	89.677	270.060	267.419	266.781
	3	13.068	12.492	11.596	95.692	95.014	91.463	280.796	279.873	275.822
	4	10.629	10.352	10.380	74.539	75.415	71.564	233.582	232.409	227.653
	5	9.105	8.786	9.708	53.995	52.729	51.745	161.878	160.367	159.949
	6	8.510	8.526	8.944	51.848	51.464	49.434	158.552	161.631	156.958
	7	7.974	8.091	8.581	51.119	50.244	47.003	155.530	157.629	152.716
	8	7.531	7.694	8.499	45.775	45.523	43.527	140.609	136.886	135.961
Std. Dev.	1	8.592	8.388	9.930	36.101	34.975	36.782	72.598	68.194	70.975
	2	6.909	6.785	6.912	25.819	26.291	26.809	52.447	50.903	50.850
	3	5.979	6.087	5.937	21.968	21.464	22.748	42.166	42.498	42.728
	4	5.415	4.970	5.480	21.536	21.471	22.203	45.301	44.470	46.345
	5	4.695	4.633	4.937	19.439	19.500	19.652	42.855	42.711	41.491
	6	4.410	4.335	4.764	18.154	17.815	18.554	41.853	40.436	40.459
	7	4.178	4.174	4.491	17.429	17.507	17.497	38.837	37.615	38.141
	8	3.866	4.091	4.371	17.012	17.129	16.889	37.150	36.239	37.578

 Table 1. Summary of simulation results.



Fig. 6. Ranging errors and calculated ranging errors. (a) σ is 1 dBm (b) σ is 3 dBm

summary of the simulations.

Finally, Fig. 6 compares the estimated distance estimation error according to measurement errors presented in Eq. (13) and simulation-based distance calculation errors. This comparison confirmed that despite the same statistical error size, the distance calculation error increased gradually according to distance, and that distance calculation errors in the simulations had a similar size and pattern to that of calculated distance errors through the derived equation.

The simulation results confirmed that the position of base station can be estimated based on measurement data, and the accuracy of the positioning was affected by the range of measurement data, DOP, the accuracy of GPS-based position information, and the errors of the wireless signal included in the RSRP. In particular, the errors included in the RSRP may generate large errors in distance calculation if the error is not compensated, and thus the estimated position information of the base station may have large errors. In urban areas, not only signal noise but also errors caused by the NLOS and multi-path signals are included in the RSRP, and thus are expected to significantly affect the position estimation of base stations. Thus, it is important to set up and adjust the path loss model properly in urban areas for accurate position estimation of base stations. This will also considerably affect the accuracy of DB construction of unmeasured areas which will be constructed based on the path loss model.

4. CONCLUSIONS

In this paper, we propose a technique for estimating the location of base station based on measurement data for wireless positioning for emergency rescue. It shows that the RSRP of wireless signals can be stored together with the position obtained through the GPS receiver and the position of the base station can be estimated based on this.



The error analysis and simulation results showed that the measurement collection area, DOP, GPS-based positioning accuracy and signal errors affected the accuracy of the positioning of the base station, and the size of signal errors was the largest factor to affect the accuracy. In addition, it was confirmed that the accuracy of positioning could be improved by acquiring evenly measurements in the service area. Through this, it can be expected to contribute to researching the path loss model and the setup of DB construction scenario in the service area.

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AUTHOR CONTRIBUTIONS

Seong Yun Cho contributed to the conceptualization of the idea, implemented the algorithm and wrote the original draft of the manuscript as a project administrator. Chang Ho Kang assisted with the research and reviewed the manuscript. All authors discussed the proposed approach and results.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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